SAMPLE PAPER 2 - Answer

First year Higher Secondary Examination
PART III
MATHEMATICS (SCIENCE)
Maximum: 80 (Scores)

1. a) $\left\{x : \frac{n}{n+1}, n \in N, n \le 6\right\}$

   b) $A \cup B = B$

   c) A – Set of individuals exposed to chemical C_1

   B – Set of individuals exposed to chemical C_2

   $n(U) = 200; n(A) = 120; n(B) = 50$

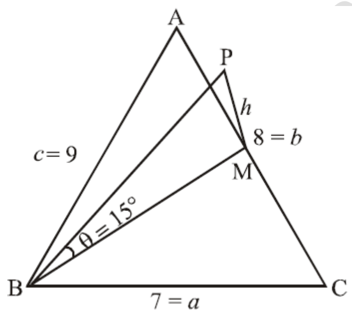
   $n(A \cap B) = 30$

   No. of individuals exposed to chemical C_2 but not $C_2 = n(A-B)$

   $= n(A) - n(A \cap B)$

   $= 120 - 30 = 90$

2.



Using cosine rule, $cosC = \frac{a^2+b^2-c^2}{2ab} = \frac{7^2+8^2-9^2}{2(7)(8)} = \frac{2}{7}$

Now, $BM^2 = BC^2 + CM^2 - 2BC.CM.cosC = 7^2 + 4^2 - 2(7)(4) \times \frac{2}{7} = 49$

$\therefore BM = 7$

$In\ rt.\Delta BMP,\ \tan 15 = \frac{h}{7}$

$\therefore h = 7\tan 15 = 7\left(2-\sqrt{3}\right)m$ || $\tan 15 = 2-\sqrt{3}$, we have proved earlier.

3. $-15-8i$

Let $\sqrt{8-i6} = x+iy$ .... (1)

Squaring we have,

$8-i6 = (x+iy)^2 = x^2 + i2xy + i^2y^2 = \left(x^2-y^2\right) + i2xy$

Equating the real and imaginary parts, we have

$x^2 - y^2 = 8$ ....(2)

$2xy = -6$ ....(3)

We know that $\left(x^2+y^2\right)^2 = \left(x^2-y^2\right)^2 + 4x^2y^2 = (8)^2 + (-6)^2 = 64+36 = 100$

$\therefore x^2 + y^2 = \sqrt{100} = 10$ ....(4)

(2) +(4) we have,

$$x^2 - y^2 = 8$$
$$x^2 + y^2 = 10$$
$$-----------$$
$$2x^2 = 18$$
$$x^2 = 9 \Rightarrow x = \pm 3$$

In (4), we have, $3^2 + y^2 = 10 \Rightarrow y^2 = 10 - 9 = 1 \Rightarrow y = \pm 1$

Since $2xy = -6$,

When $x = 3, y = -1$ and when $x = -3, y = 1$

$3 - i$ and $-3 + i$ are the square roots.

4. Let the mark in the first exam $= x$

Mark in the second exam $= x + 5$

Mark in the third exam = $x + 10$

Given that $\dfrac{x + x + 5 + x + 10}{3} \geq 80$

$$\frac{3x+15}{3} \geq 80$$
$$x \geq 21.7$$
$$3x + 15 \geq 3 \times 80$$
$$3x \geq 240 - 15$$
$$3x \geq 225$$
$$x \geq \frac{225}{3}$$
$$x \geq 75$$

Minimum mark Arathi should get in the first exam = 75.

5. a) $s = \{6, (1,6), (1,1,6), ..., (2,6), (2,1,6), ...\}$

b) $n(s) = 36$

doublets are (1,1), (2,2),(3,3), (4,4),(5,5) and (6,6)

$\therefore P$ (getting a doublet) $= \dfrac{6}{36} = \dfrac{1}{6}$

6. a) Ans : C

b) $2a = 26 \Rightarrow a = 13$

foci $= (\pm 5, 0) \Rightarrow c = 5$

But $c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 13^2 - 5^2 = 144$

$\therefore$ equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{169} + \frac{y^2}{144} = 1.$

7. $\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx} = \frac{0}{0}$

$$\lim_{x \to 0} \frac{\frac{\sin ax}{ax} \times ax + bx}{ax + \frac{\sin bx}{bx} \times bx} = \lim_{x \to 0} \left[\frac{\frac{\sin ax}{ax} \times a + b}{a + \frac{\sin bx}{bx} \times b} \right]$$

$as\, x \to 0, ax \to 0, bx \to 0$

$= \frac{1 \times a + b}{a + 1 \times b}$

$= \frac{a+b}{a+b} = 1$

8. a) $\frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - 1^2}{(1.1 - 1)} = \frac{(1.1+1)(1.1-1)}{(1.1-1)} = 1.1 + 1 = 2.1$

b) Here $B \cap C = \phi$

$A \times B = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

$A \times C = \{(1,5),(1,6),(2,5),(2,6)\}$

$B \times D = \{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),$
$(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\}$

i) $A \times (B \cap C) = A \times \phi = \phi$ and $(A \times B) \cap (A \times C) = \phi$

[$\because A \times B$ and $A \times C$ have no common ordered pair]

$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$, (Each $= \phi$)

ii) Since every member (i.e. ordered pair) of $A \times C$ is member (i.e. ordered pair) of $B \times D$,

$\therefore A \times C$ *is subset of* $B \times D$.

9. $P(n): 1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2} \right]^2$, $n \in N$

a) *Let* $P(1): 1^3 = \left[\frac{1(1+1)}{2} \right]^2 \Rightarrow 1 = \left[\frac{1(2)}{2} \right]^2 \Rightarrow 1 = 1$

Hence, $P(1)$ *is true.*

b) *Assume that* $P(k)$ *be true.*

$P(k): 1^3 + 2^3 + 3^3 + ... + k^3 = \left[\frac{k(k+1)}{2} \right]^2$

To prove that $P(k+1)$ *is true.*

$$P(k+1):1^3+2^3+3^3+...+k^3+(k+1)^3=\left[\frac{(k+1)(k+2)}{2}\right]^2$$

$$\Rightarrow P(k)+(k+1)^3=\left[\frac{(k+1)(k+2)}{2}\right]^2$$

$$\Rightarrow\left[\frac{k(k+1)}{2}\right]^2+(k+1)^3=\left[\frac{(k+1)(k+2)}{2}\right]^2$$

$$Now,\ LHS=\left[\frac{k(k+1)}{2}\right]^2+(k+1)^3$$

$$=(k+1)^2\left[\frac{k^2}{4}+(k+1)\right]$$

$$=(k+1)^2\left[\frac{k^2+4k+4}{4}\right]=(k+1)^2\left[\frac{(k+2)^2}{4}\right]$$

$$=\left[\frac{(k+1)(k+2)}{2}\right]^2=RHS$$

Hence, $P(k+1)$ *is true.*

Hence, $P(n)$ *is true for all* $n\in N$.

10\. a) $(5-3i)^3=5^3-3\times5^2(3i)+3\times5(3i)^2-(3i)^3$

$$=125-225i+135i^2-27i^3$$
$$=125-225i+135(-1)-27(-i)$$
$$=125-225i-135+27i$$
$$=-10-198i$$

b) $-x^2+x+2=0$

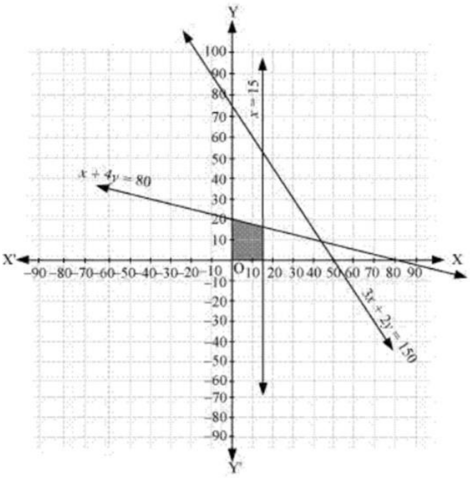
$a=-1;b=1;c=2$

$$D=b^2-4ac=(1)^2-4(-1)(2)$$
$$=1-8=-7<0$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$=\frac{-1\pm\sqrt{-7}}{2\times-1}=\frac{-1\pm i\sqrt{7}}{-2}=\frac{1\mp i\sqrt{7}}{2}$$

11\.



The shaded region is the solution region.

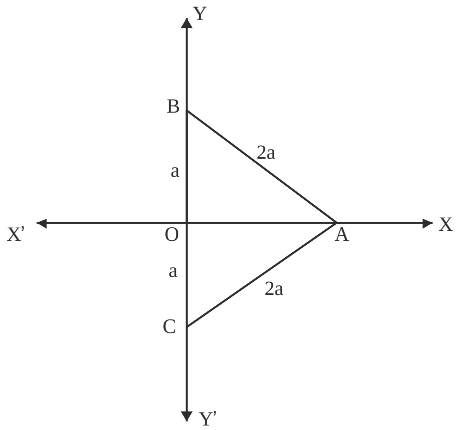
12. a) Here unit place can filled in three ways (i.e. by 2, 4, 6), whereas tens and hundred place can be filled in 6 ways

$\Rightarrow$ Total number of 3-digit even numbers = (3) (6) (6) =108.

b) Out of available nine courses, two are compulsory. Hence the student is free select 3 courses out of 7 remaining courses. Then the number of ways of selecting 3 courses out of 7 courses

$$= C(7,3) = \frac{7!}{3!7!-3!} = \frac{7!}{3!4!} = 7 \times 5 = 35\, ways.$$

13. a)



Let ABC be the given equilateral triangle with side 2*a*.

Accordingly, AB = BC = CA = 2*a*

Assume that base BC lies along the *y*-axis such that the mid-point of BC is at the origin.

i.e., BO = OC = *a*, where O is the origin.

Now, it is clear that the coordinates of point B are (–*a*,0) and C are (a,0), while the coordinates of point.

Using Pythagoras theorem to ΔAOC, we obtain

$AC^2 = OA^2 + OC^2 \Rightarrow (2a)^2 = OA^2 + a^2 \Rightarrow 4a^2 - a^2 = OA^2$

$\Rightarrow OA^2 = 3a^2 \Rightarrow OA = \sqrt{3}a$

$\therefore$ co – ordinates of A are $(\pm\sqrt{3}a, 0)$. Thus, the vertices of the given equilateral triangle are (0, *a*), (0, –*a*), and $(\sqrt{3}a, 0)$ or (0, *a*), (0, –*a*), and $(-\sqrt{3}a, 0)$

b) Any line through (-3,5) is $y - 5 = m(x - (-3)) = m(x+3)$

Slope of line joining (2,5) and (-3,6) $= \frac{6-5}{-3-2} = \frac{1}{-5}$

$\therefore$ slope of a line $\perp$ to it = 5 $\therefore$ reqd. line is $y - 5 = 5(x+3) \Rightarrow 5x - y + 20 = 0$

14. a) centre = $(-g, -f) = (4, -5)$

$$r=\sqrt{(-4)^2+5^2--12}$$
$$=\sqrt{16+25+12}$$
$$=\sqrt{53}$$

b) Equation is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$(x-a)(x-0)+(y-0)(y-b)=0$

$x^2-ax+y^2-by=0$

$x^2+y^2-ax-by=0$

15. a) The equation of line making intercepts a and b on the axes is $\frac{x}{a}+\frac{y}{b}-1=0$ (intercept from)…(i)

Given $p=$ perpendicular distance from (0,0) on (i)

$\therefore p=\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$ Squaring we get $p^2=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$ or $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

b) Three lines are said to be concurrent, if they pass through a common point, i.e., point of intersection of any two lines lies on the third line. Here given lines are

$2x+y-3=0 \quad ...(1)$

$5x+ky-3=0 \quad ...(2)$

$3x-y-2=0 \quad ...(3)$

Solving (1) and (3) by cross- multiplication method, we get,

$\frac{x}{-2-3}=\frac{y}{-9+4}=\frac{1}{-2-3}$ or $x=1, y=1$

Therefore, the point of intersection of two lines is (1,1). Since above three lines are concurrent, the point (1,1) will satisfy equation (2) so that

$5.1+k.1-3=0\Rightarrow k=-2.$

16. a) (–3,1,2) – 2^{nd} octant

(–3,1,–2) – 6^{th} octant

b) Let A $(0,y,0)$ be a point on y- axis having distance $5\sqrt{2}$ from $P(3,-2,5)$

$\Rightarrow \quad |AP|=5\sqrt{2}\Rightarrow\sqrt{(3-0)^2+(-2-y)^2+(5-0)^2}=5\sqrt{2} \quad ....(i)$

$\Rightarrow 9+4+4y+y^2+25=50 \quad (on\ squaring\ (i))$

$\Rightarrow y^2+4y-12=0\Rightarrow(y+6)(y-2)=0\Rightarrow y=-6,2$

$\therefore$ $A(0,-6,0)$ and $A(0,2,0)$ are the required points.

17. a) If you do not do all the exercises in this book, you will not get an a grade in the class.

b) Let us assume that $\sqrt{2}$ be rational.

$\therefore$ $\sqrt{2}=\frac{a}{b}$, where a and b are co-prime. i.e., a and b have no other common factors except 1.

Then $2b^2=a^2\Rightarrow 2$ divides a.

$\therefore$ there exists an integer 'k' such that $a=2k$

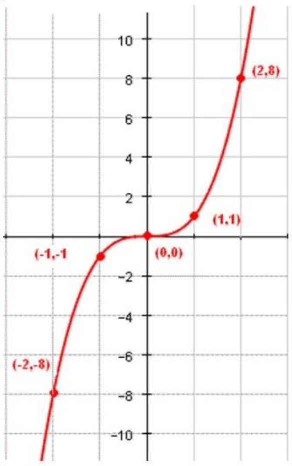
$\therefore$ $2b^2=4k^2\Rightarrow 4k^2=2b^2=2k^2\Rightarrow b^2\Rightarrow 2$ divides b.

i.e., 2 divides both a and b, which is contradiction to our assumption that a and b have no common factor.

$\therefore$ our supposition is wrong.

$\therefore$ $\sqrt{2}$ is an irrational number.

18. a)



b) (iii) $n(b)^{n(A)} = 3^2$

19. a) $\sec x = \frac{13}{5}$

$\cos x = \frac{5}{13}$

$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$

In Q IV, $\sin x$ is –ve; $\therefore \sin x = \frac{-12}{13}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{12}{13}}{\frac{5}{13}} = -\frac{12}{5}$

$\operatorname{cos}ecx = \frac{1}{\sin x} = \frac{-13}{12}$

$\cot x = \frac{1}{\tan x} = \frac{-5}{12}$

b) We know that $3x = 2x + x$

$\therefore \tan 3x = \tan(2x + x)$

$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

$\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan$

$\Rightarrow \tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

c) wkt $\tan^2 x = \dfrac{1-\cos 2x}{1+\cos 2x}$

$$\therefore \tan^2 \frac{\pi}{8} = \frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}$$

$$= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$= \frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$$

$$= \frac{(\sqrt{2}-1)^2}{2-1} = (\sqrt{2}-1)^2$$

$$\therefore \tan\frac{\pi}{8} = \sqrt{2}-1$$

20. a) This is not a G.P., however, we can relate it to a G.P by writing the terms as

$$S_n = 7+77+777+7777+....to\, n\, terms$$

$$= \frac{7}{9}[9+99+999+9999+...to\, n\, term]$$

$$= \frac{7}{9}[(10-1)+(10^2-1)+(10^3-1)+(10^4-1)+....n\, terms]$$

$$= \frac{7}{9}[(10+10^2+10^3+...n\, terms)-(1+1+1...n\, terms)]$$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1}-n\right] = \frac{7}{9}\left[\frac{10(10^n-1)}{9}-n\right]$$

b) Let T_n denote the nth term of the given series

$$T_n = (n^{th}\ term\ of\ 3,5,7,...)(n^{th}\ term\ of\ 1^2,2^2,3^2,...)$$

$$= [3+(n-1)(2)].n^2 = (2n+1)n^2 = 2n^3+n^2$$

$$\therefore S_n = 2\sum n^3 + \sum n^2 = 2\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)}{2}\left[n(n+1)+\frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+5n+1}{3}\right] = \frac{n(n+1)(3n^2+5n+1)}{6}$$

21. a) Here $n=10$ is even.

Middle term $\dfrac{10}{2}+1 = 6^{th}$ term

$$T_6 = T_{5+1} = {}^{10}C_5\left(\frac{x}{3}\right)^5(9y)^5 = (252)\left(x^5y^5\right)\left(\frac{9}{3}\right)^5 = (252)(243)x^5y^5 = 61236x^5y^5$$

b) We have $T_{r+1} = {}^{18}C_r\left(\sqrt[3]{x}\right)^{18-r}\left(\frac{1}{2\sqrt[3]{x}}\right)^r$

$$= {}^{18}C_r\, x^{\frac{18-r}{3}}.\frac{1}{2^r.x^{\frac{r}{3}}} = {}^{18}C_r\,\frac{1}{2^r}.x^{\frac{18-2r}{3}}$$

Since we have to find a term independent of x, i.e., term not having x, so take $\frac{18-2r}{3}=0$. We get $r=9$. The required term is ${}^{18}C_9\times\frac{1}{2^9}$.

22. a) Let $f(x) = x\sin x$

$$f(x+h) = (x+h)\sin(x+h)$$

$$f'(x) = \lim_{x\to 0}\left[\frac{(x+h)\sin(x+h)-x\sin x}{h}\right]$$

$$= \lim_{x\to 0}\left[\frac{x\sin x(\cos h-1)+x\cos x\sin h+h(\sin x\cos h+\cos x\sin h}{h}\right]$$

$$= \lim_{x\to 0}\left[x\sin x\times\frac{(\cos h-1)}{h}+x\cos x\times\frac{\sin h}{h}+\sin x\cos h+\cos x\sin h\right]$$

$$= x\sin x\times 0+x\cos x\times 1+\sin x\cos 0+\cos x\sin 0$$

$$= x\cos x+\sin x \qquad [\cos 0 = 1,\ \sin 0 = 0]$$

$$= 2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)$$

b) $y = \dfrac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$

$$\frac{dy}{dx} = \cos\left(\frac{\pi}{4}\right)\times\frac{d}{dx}\left(\frac{x^2}{\sin x}\right)$$

$$= \cos\left(\frac{\pi}{4}\right)\times\frac{\sin x.\frac{d}{dx}\left(x^2\right)-x^2.\frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{1}{\sqrt{2}}\times\frac{\sin x(2x)-x^2\cos x}{\sin^2 x}$$

$$= \frac{2x\sin x-x^2\cos x}{\sqrt{2}\sin^2 x}$$

23. a) we have

x_i	f_i	$f_i x_i$	$\lvert x_i - 50\rvert$	$f_i \lvert x_i - 50\rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	$N = 80$	$\sum f_i x_i = 4000$		$\sum f_i \lvert x_i - 50\rvert = 1280$

Mean = $\dfrac{\sum f_i x_i}{N} = \dfrac{4000}{80} = 50$

M.D(mean) $= \dfrac{\sum f_i \lvert x_i - 50\rvert}{N} = \dfrac{1280}{80} = 16$

b) We first make the data continuous by making classes as below:
32.5 - 36.5, 36.5 - 40.5, 40.5 - 44.5 44.5 - 48.5, 48.5 - 52.5

Diameter (in mm)	Mid – value (x_i)	No. of inches (f_i)	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$	$f_i u_i^2$
32.5 - 36.5	34.5	15	-2	-30 $\big] -47$	60
36.5 - 40.5	38.5	17	-1	-17	17
40.5 - 44.5	42.5	21	0	0	0
44.5 - 48.5	46.5	22	1	22 $\big] 72$	22
48.5 - 52.5	50.5	25	2	50	100
		$N = 100$		$\sum f_i u_i = 25$	$\sum f_i u_i^2 = 199$

Let assumed mean $a = 42.5$, h = 4

$$\therefore Mean, \bar{x} = a + \left(\sum \frac{f_i u_i}{N}\right) \times h = 42.5 + \frac{1}{100}(25) \times 4 = 42.5 + 1 = 43.5cm$$

$$\therefore \ Mean = 43.5cm$$

$$\text{Variance, } \sigma^2 = \left[\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2\right] \times h^2 = \left[\frac{199}{100} - \left(\frac{25}{100}\right)^2\right] \times 16$$

$$= 16(1.99 - .0625) = 16[1.9275] = 30.84$$

$$\therefore S.D = \sqrt{\sigma^2} = \sqrt{30.84} = 5.55$$

24. a) $P(a1aces) = \dfrac{{}^{4}C_4}{{}^{52}C_4} = \dfrac{1}{270725}$

b) $P\left(at\ most\ 2ace\ \right)=P\left(no\ ace\right)+P\left(1ace\right)+P\left(2\ aces\right)$

$$=\frac{^{48}C_4}{^{52}C_4}+\frac{^4C_1\times^{48}C_3}{^{52}C_4}+\frac{^4C_2\times^{48}C_2}{^{52}C_4}.$$

c) P(atleast 2 aces) = P(2 aces)+P(3 aces)+P(4 aces)

$$=\frac{^4C_2\times^{48}C_2}{^{52}C_4}+\frac{^4C_3\times^{48}C_1}{^{52}C_4}+\frac{^4C_4}{^{52}C_4}.$$

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